

# *Solutio Theorematum*

by Adam Adamandy Kochański – Latin text with annotated  
English translation

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*Translator's note:* The Latin text of *Solutio theorematum* presented here closely follows the original text published in *Acta Eruditorum* [1]. Punctuation, capitalization, and mathematical notation have been preserved. Several misprints which appeared in the original are also reproduced unchanged, but with a footnote indicating correction. Every effort has been made to preserve the layout of original tables. The translation is as faithful as possible, often literal, and it is mainly intended to be of help to those who wish to study the original Latin text. In the appendix, all propositions from Euclid's *Elements* mentioned in the text are listed in both Latin and English version.

*SOLUTIO THEOREMATUM*  
*Ab illustri Viro in Actis hujus*  
*Anni Mense Januario, pag. 28.*  
*propositorum, data ab Adamo*  
*Adamando Kochanski S.J.*  
*quondam Pragensi Mathematico.*

DUPLICATIONEM Trigoni Isogoni, citra  
Proportiones demonstrandam P. Sigismundus  
Hartman<sup>1</sup> e Soc. Jesu, publico Programma-  
te proposuerat, istudque Schediasma mihi pro  
veteri necessitudine transmiserat e Bohemia  
in Poloniam. Reposui humanissimo Autho-  
ri solutiones fere vicens, ab eo sic proba-  
tas, ut una cum aliis, aliunde ad se missis,  
in lucem daturus fuisset, si Parcae Viro tanto  
pepercissent.

Cum vero his primum diebus in manus  
meas venerint Acta Eruditorum Lipsiensia, &  
in his Solutio problematis istius Hartmannia-  
ni, a quodam illustri viro data, & ad P. Cop-  
pilium<sup>2</sup>, defuncti in Mathematici munere suc-  
cessorem, quodammodo directa, quasi is edi-

*SOLUTION OF THE THEOREM*  
*proposed by an illustrious man in*  
*this year's January issue of Acta*  
*on p. 28, given by Adam*  
*Adamandy Kochanski SJ, once*  
*mathematician of Prague.*

Fr. Sigismundus Hartman from Soc. of Je-  
sus proposed in a public program the problem of  
DUPLICATION of an equilateral triangle with-  
out the use of proportions, and conveyed this  
question to me from Bohemia to Poland, on the  
account of old friendship. I returned to the kind-  
est author perhaps twenty solutions, by him thus  
examined, so that one of these, together with  
others, send to him from elsewhere, was to be  
put to light, if only Fate had spared the man so  
great.

When, however, those days Acta Eruditorum  
of Leipzig came into my hands, and in them so-  
lution of this problem of Hartman, given by a  
certain illustrious man, directed to Fr. Copil-  
lus, successor of the deceased in the mathemat-

<sup>1</sup>Sigismundus Ferdinandus Hartmann SJ (1632–1681) – Bohemian Jesuit and mathematician, professor of the University of Prague.

<sup>2</sup>Matthaeus Coppelius SJ (1642–1682) – Bohemian Jesuit and mathematician, author of books on mechanics.

25 tioni posthumae operum Hartmanni, maxime-  
 que *Protei Geometrici*, ab eo nuper promissi  
 incumberet ; cum tamen ab obitu Authoris  
 elaboratum nihil, sed prima solum Operis li-  
 30 neamenta reperta fuisse mihi ab Amicis nun-  
 ciatum fuerit ; Eam ob rem non aegre latu-  
 rum spero P. Coppilium, si dum illum alio in  
 opere fructuose versari intelligo, hic ejus par-  
 tes occupare ausus fuero: non enim it temere,  
 aut praesidenter egisse videbor, sed veluti ju-  
 re quodam antiquitatis ; quod videlicet ante  
 35 illum in eodem Matheseos Pragensis pulvere  
 quendam Professor fuerim versatus; adeoque  
 prior tempore, licet non eruditione.

Dilatis autem in aliud tempus meis illis  
 Duplicationum Particularium, & Universa-  
 40 lium Solutionibus, una cum Pythagoricae  
 nova, ac multiplici Demonstratione, cete-  
 risque meis considerationibus Geometricis,  
 usque dum ultroneam Typographi, vel Bi-  
 bliopolae cujuspian humanitatem invenerint;  
 45 suffecerit hoc loco strictim ea persequi, quae  
 pertinent ad geminum illud Theorema Geo-  
 metricum, quod Anonymus ille Problematis  
 Hartmanniani  $\delta\epsilon\acute{\iota}\chi\tau\eta\varsigma$  loco sup. memorato  
 proposuit.

50

## ARTICULUS I.

Circa primum illorum Theorematum con-  
 sideranda veniunt sequentia. Inprimis dissen-  
 tire me in eo ab *Illustri Viro*, quod is existimet  
 55 Theorema Pythagoreum continuari in Sphae-  
 ra, Pyramidem Rectangulam circumscriben-  
 te; cum nec Pythagoras suum illud Orthogo-  
 nium, tanquam circulo inscriptum considera-  
 verit, nec, si universaliter agamus de poten-  
 tiis cujusvis Trianguli, haec consideratio pro-  
 60 prie ad Circulum pertinere videatur, sed po-  
 tius ad Parallelogrammum universim ; quan-  
 do videlicet istud Diametro sua sectum concipitur  
 in duo Triangula aequalia, eaque Ortho-  
 65 gonia, Amblygonia, vel Oxygonia, pro diver-  
 sitate parallelogrammi; Harum enim Diame-

ical office, and arranged in a certain way, as if  
 he was leaning towards posthumous edition of 25R  
 Hartman's works, especially *Protei Geometrici*,  
 recently promised by him; when still from the  
 death of the Author nothing has been worked  
 out, but friends announced to me that only the  
 preliminary outline of the Works has been pro- 30R  
 duced; therefore I hope that Fr. Copillus is not  
 offended if, while he, as I understand, is busy  
 with other fruitful works, I dared to assume his  
 role: this will not, indeed, seem to be acting  
 rashly, or daringly, but by a certain old right; 35R  
 because evidently I used to dwell in the same  
 dust of Prague as Professor of Mathematics<sup>3</sup>; in-  
 deed, preceding [Fr. Hartman] in time, but not  
 in erudition.

Leaving for another time my particular and 40R  
 general solutions of the Duplication, one with a  
 novel Pythagorean proof, others based on my Ge-  
 ometrical considerations, until they find a willing  
 printer or a kind publisher; it will suffice in this  
 place to pursue only what pertains to these two 45R  
 Geometrical Theorems, which the Anonymus ex-  
 hibitor put forward in the aforementioned place.

## ARTICLE I.

The following considerations revolve around 50R  
 the first of these theorems. First of all I disagree  
 with the *Illustrious Man* in his opinion that  
 the Pythagorean Theorem extends to a sphere  
 circumscribed around a right-angled pyramid;  
 when neither Pythagoras thought of his right 55R  
 triangle as if it was inscribed in a circle, nor,  
 if we talk in general about properties of an  
 arbitrary triangle, does such considerations  
 seem to pertain specifically to a circle, but  
 rather to parallelogram in general; when clearly 60R  
 this [parallelogram] is understood to be cut  
 by its diameter [i.e., diagonal] into two equal  
 triangles, either right-angled or obtuse-angled  
 or acute-angled, according to diversity of paral-

<sup>3</sup>Kochański stayed in Prague from 1670 to 1672, lecturing in mathematics and (probably) moral philosophy.

trorum Potentiae cum suis lateribus comparantur 47. *Primi*, nec non 13, & 13. *Secundi Elem.* Universalius autem hoc ipsum consideratur 31 *Sexti* saltem quoad orthogonium Triangulum ; nam quoad reliqua, videri possunt ea, quae demonstrat Clavius e Pappo, in *Scholio ad 47. Primi, ad finem.*

Quamobrem non ineleganter fluente Analogia, nobis dicendum videtur, Pyramides Triangulares Orthogonias, Amblygonias, & Oxygonias, cum suis laterum, ac diameterum Potentiis, immediate quidem reduci oportere ad Prismata sua, bases triangulares habentis, quorum Pyramides illae sunt partes tertiae *per Prop. 7. Duodecimi Elem.* Haec ipsa vero Prismata, tanquam partes revocantur ad totum Parallelepipedum, cuius sunt medietates. Ista porro relationes partium ad sua Tota intelligi volumus de ordine *Doctrinae* potius, quam *Naturae*, constat enim, Triangulum esse prius, ac simplicius Parallelogrammo, non minus ac Pyramidem Tetrahedram Prismaticam, vel Parallelepipedo: Hinc  $\delta\omicron\gamma\mu\alpha\tau\iota\chi\tilde{\omega}\varsigma$  sperando, & ipsae Linearum Potentiae, non Triangulis aequilateris, sed Quadratis, omnium consensu taxantur, licet illa sint istis priora, magisque simplicia.

Quamvis autem Pyramidum Polyhedrarum aliae inscribi possint Sphaeris, aliae vero Sphaeroidibus, ac tum Potentiae laterum conferri cum Diametro corporis circumscribentis: eadem tamen Pyramides adhuc secari poterunt in Tetrahedras, atque ita Prismatibus suis, ac tum Parallelepipedis veluti postliminio quodam restitui. Si quis nihilominus omnia Trilatera a Quadrilateris, omnia Tetrahedra a Pentahedris & Hexahedris emancipare contenderit cum eo nequaquam cruento Marte dimicabimus.

lelograms; Powers [squares] of these diameters [diagonals] are compared with powers of their [parallelograms] sides in prop. 47 of the *first book of Elements* as well as in and prop. 13 and prop. 13<sup>4</sup> of the *second book*. This is considered more generally in prop. 31 of the *sixth book*, as long as the right triangle is considered. On the other hand, cases which remain can be seen as those demonstrated by Clavius following Pappus *at the end of commentary to prop. 47*.<sup>5</sup>

It seems we need to state an elegantly flowing analogy, that it is right to put triangular prisms, right-angled as well as obtuse-angled and acute-angled, with powers [squares] of their sides and diameters, into their respective prisms, having triangular bases, of which these prisms constitute third parts [by volume], by *prop. 7 of the twelfth book*. Such prism are in truth recalled just as if they were parts of a complete parallelepiped, of which they are halves. Hereafter we want these relations of parts to their wholes to be understood from *Principles* rather than from *Nature*, as it is evident that the triangle is prior to and simpler than a parallelogram, not less as the tetrahedral pyramid is prior to and simpler than a prism, or parallelepiped: from there observing the thing dogmatically, powers of own lines taken together are valued not with equilateral triangles, but squares, although those [triangles] are prior to these [squares], and much simpler.

Although some polyhedral pyramids could be inscribed in spheres, and others in spheroids [i.e., ellipsoids of revolution], and then powers of their sides could be matched against diameters of circumscribed bodies: besides, the same pyramids could be divided into tetrahedrons, and therefore also brought into their own prisms, and then parallelepipeds, as if by the right to return home. None the less, if somebody wanted to alienate all trilaterals from quadrilaterals, all tetrahedra from pentahedra and hexahedra, we will by no means spill blood in a fight with him.

<sup>4</sup>Obviously, this should be “prop. 12. and prop. 13”.

<sup>5</sup>Pappus builds parallelograms on sides of an arbitrary triangle, cf. p. 366 of [2].

## PROPOSITIO I. Theorema.

In omni Pyramide rectangula, tria  
 110 Quadrata laterum, Angulum rectum  
 in vertice comprehendentium, aequalia  
 sunt Quadrato Diametri totius Paral-  
 lelepipedi aequae alti, Pyramidem illam  
 complectentis.

Sit Pyramis ABC rectangula ad verticem  
 115 D. sive jam sit *Aequilatera*, prout in Cubo *Fi-*  
*gura* I. sive *Isosceles*, ut est in Fig. II Pa-  
 rallelepipedo, supra basin quadratam ADCF  
 assurgentis ; sive demum *Scalena*, qualem ex-  
 hibet in Fig. III. solidum rectangulum, supra  
 120 basin ADCF altera parte longiorem, erectum.

Dico, tria Quadrata DA. DB, DC aequari  
 Quadrato Diametri AE, per oppositos solidi  
 angulos incedentis. Nam primam in Trian-  
 gulo ADB angulus D rectus est, ex hypoth.  
 125 Igitur Qu. lateris AB aequatur (*47. I. Elem.*)  
 Quadratis AD. DB duorum laterum datae Py-  
 ramidis. Deinde Triangulum pariter ABE rec-  
 tangulum est ad B. (id ostendi potest *per 4.*  
*Undecimi*) Quocirca Quadratum AE aequa-  
 130 bitur Quadrato AB, hoc est duobus DA. DB,  
 & insuper Quadrato BE, hoc est ipsi aequali  
 DC, quod est tertium latus datae Pyramidis  
 ABCD.

Tria igitur omnis Pyramidis rectangulae  
 135 latera, potentia aequantur Diametro Paral-  
 lelepipedo Pyramidem continentis, q. e. d.

## Corollarium I.

Colligitur hinc, in omni Prismate rectangulo  
 140 ACDBGA, cui basis est Orthogonium Trian-  
 gulum ADC cujus Parallelogrammi rectanguli  
 AGECE, quod angulos D. & B. rectos subten-  
 dit, Diametrum AE. Potentia aequari iisdem  
 tribus lateribus Pyramidis rectangulae AB-

<sup>6</sup>Having three equal sides.

<sup>7</sup> $AB^2 = AD^2 + DB^2$ ,  $AE^2 = AB^2 + BE^2 = AD^2 + DB^2 + BE^2 = AD^2 + DB^2 + DC^2$ .

<sup>8</sup>i.e., sum of their square is equal to the square of...

## PROPOSITION I. Theorem.

In every right-angled pyramid, sum of  
 squares of three sides coming from the  
 110R vertex embracing the right angle is equal  
 to the square of the diameter of the  
 complete parallelepiped of equal height,  
 encompassing this pyramid.

Let the pyramid ABC be right-angled at the  
 115R vertex D, whether *equilateral*, as in the cube of  
 Fig. I, or *isosceles*, as in the parallelepiped of  
 Fig. II, rising over the square basis ADCF, or  
 finally *sceles*<sup>6</sup>, as in the rectangular solid erected  
 over the basin ADCF elongated in the other di-  
 120R rection, as displayed in Fig. III.

I say that three squares of DA, DB, and DC  
 are equal in sum to the square of the diame-  
 ter AE, stretched between opposite angles of the  
 solid. For, first of all, in the triangle ADB, the  
 125R angle D is right, by hypothesis. Therefore the  
 square of AB is equal (by *prop. 47 of Elem. I*)  
 to the sum of squares of two sides AD and DB  
 of the given pyramid. Next also triangle ABE is  
 right-angled at B (this can be shown by *prop. 4*  
 130R *of the eleventh book*), on account of which square  
 of AE is equal to the square of AB, that is sum of  
 squares of DA and DB, plus square of BE, which  
 itself is equal to DC, the third side of the given  
 pyramid ABCD.<sup>7</sup> 135R

Therefore three sides of any rectangular pyra-  
 mid are equal in power<sup>8</sup> to the diameter of the  
 parallelepiped enclosing the pyramid, Q.E.D.

## Corollary I.

One obtains from there that in all rectangular  
 prisms ACDBGA, whose base is a right triangle  
 ADC, square of the diameter AE of the right-  
 angled parallelogram AGECE, which extends be-  
 145R low right angles D and B, is equal to sum of  
 squares of three sides of the right-angled pyramid

145 CD: eo quod ipsa DE sit eadem omnino cum  
Diametro totius Solidi GC.

### Corollarium II.

150 Hinc quoque manifestum est, Diametrum  
Sphaerae, quae Pyramidi rectanguli ABCD  
circumscripita est, aequari potentia tribus  
lateribus ejusdem Pyramidis, rectos angulos  
constituentibus in vertice D. Nam cum omni  
155 solido rectangulo Sphaera circumscribi possit,  
non minus ac Circulus ejus basi rectangulae,  
erit Diameter solidi eadem omnino quae cir-  
cumscripita Sphaerae, per ea, quae Pappus  
demonstrat *Lemmate 4. apud Clavium ad*  
*calcem Lib. 16. Elem.*

160 Si cui placuerit in simili materia ingenium  
exercere circa Pyramides Triangulares, tam  
Acutangulas, quam Obtusangulas, itemque  
mixtis in vertice angulis contentas in illis  
Potentias laterum cum Diametro totius  
165 Solidi obliquanguli comparando; id vero  
non difficulter poterit expedire ope duarum  
Propositionum, videlicet penultimae, &  
antepenultimae *Lib. Secundi Elem.*

## ARTICULUS II.

170 Circa alterum Theorema percontatur *Illustris*  
*Vir* AN sicut in Circulo unica Media Pro-  
portionalis, ita etiam insistendo Analogiae,  
in Sphaera duae Mediae inveniri possint  
175 ? Ad hanc quaestionem Respondeo I. Nec  
Antecedens Analogiae hujus tam ratum esse,  
ut absolute loquendo, Circulus duas Medias  
excludere pronunciari possit, aut debeat:  
Fieri namque potest, ut in Semicirculo ACD  
180 (*inspice Fig. 4.*) continuae sint AB. BC.  
BD. DA. cujus Problematis Geometricam  
constructionem Mathematicum peritis propo-  
no, interim vero Arithmeticum sequentibus  
numeris expono. Ponatur enim

<sup>9</sup>Clearly a misprint. Should be *AE*.

<sup>10</sup>Book XVI was a medieval addendum to *Elements*.

ABCD: consequently DE<sup>9</sup> itself would be entirely  
the same as the diameter GC of the whole solid.

### Corollary II.

150R

From this it is evident that the square of the  
diameter of the sphere circumscribed around a  
right-angled pyramid ABCD, is equal to the sum  
of squares of three sides of this pyramid forming  
the right angle at the vertex D. For while ev- 155R  
ery rectangular solid can be circumscribed by a  
sphere, as much as its rectangular base [can be  
circumscribed] by a circle, the diameter of this  
solid will be entirely the same as the diameter  
of the circumscribing sphere, as Pappus demon- 160R  
strated in *Lemma 4 in Clavius' commentary at*  
*the end of book 16<sup>10</sup> of Elements*.

If somebody would please to exercise his tal-  
ent in similar matters regarding triangular pyra-  
mids, either acute-angled or obtuse-angled, and 165R  
likewise stretching over mixed angles, comparing  
sums of squares of the sides with the diameter of  
the whole solid; this will certainly not be diffi-  
cult to obtain by the power of two propositions,  
namely the second last and the third from the 170R  
end proposition of the *second book of Elements*.

## ARTICLE II.

The Anonymous *Illustrious Man* inquires about  
another theorem, as if there exist one geomet- 175R  
ric mean in a circle, could two means be found  
in a sphere? I make first response to this ques-  
tion, that the first part of this analogy is not  
so strongly established that, absolutely speak-  
ing, it would exclude a possibility that a circle 180R  
having two means. For in fact, it can happen  
that in a semicircle ACD (*see Fig. 4*) there are  
successive lines AB, BC, BD, DA. I leave Geo-  
metric Construction of this problem to mathe-  
matical experts, and in the meanwhile I explain 185R  
Arithmetic one with the following numbers. It is  
namely assumed that diameter will be

Diameter	AD.	-	-	2 00000	00000	
erit	AB.	-	-	63534	43923.	+—
	BC.	-	-	93114	24637.	+—
	BD.	-	-	1 36465	56077.	+—

185 Unde *per 19. Septimi Elem.* erunt aequalia Rectangula

From there by *Prop. 19 of the seventh book of Elements* [the following] rectangles will be equal

DAB.	-	-	1 17068	87846	00000	00000.
CBD.	-	-	1 17068	87846	55798	69049.

Gg.

Et *per 20. ejusdem*, Quadrata mediarum aequabuntur Rectanguli sub earundem extremis.

And by *Prop. 20<sup>11</sup> of the same*, squares of means will be equal to [areas of] rectangles under their outer segments.<sup>12</sup> 190R

□BC	-	-	-	86702	62877	05305	81769.
ABD	-	-	-	86702	62877	72943	70071.
□BD	-	-	-	1 86288	49276	27056	29929.
ADBC	-	-	-	1 86228	49274	00000	00000.

190 Respondeo II. De Analoga illa nihil certi statui posse videtur: Unius enim Mediae inventio, quae Circulo tribuitur, etiam Sphaerae congruit; & inventio duarum Mediarum, hic a nobis demonstranda, aequae ad circulum, sicut  
195 & Sphaeram aptari poterit, quemadmodum ex dicendis constabit.

Secondly, I respond than nothing certain seems to possible to state about this analogy. Invention of one mean, assigned to a circle, 195R suits to sphere too. Invention of two means, demonstrated here by us, can be adapted as well to a circle as to a sphere, in what way it will agree with what has been said.

### PROPOSITIO II. Theorema.

200 In Circulo ADC a Diametro AC descripto duocantur utcunque ad peripheriam duae rectae AD DC: tum ex D cadat ad AC perpendicularis D; similiterque ex E sit ad AD normalis EF.

205 Dico in Circulo ADC, haberi Quatuor continue Proportionales, AF. AE. AD. AC. Describatur enim Diametro AD Circulus AGDE, quem EF producta secet in G, & connectantur G A. GD.

### PROPOSITION II. Theorem.

In a circle ADC traced out from a diameter AC two straight lines are drawn as far as to the perimeter: then from D a line D<sup>13</sup> perpendicular to AC line is led, and similarly from E a line EF 205R normal to AD.

I say that in the circle ACD four consecutive proportionals exist, AF, AE, AD, AC. Indeed, let a circle AGDE with diameter AD be drawn, which intersects with extension of the line EF at 210R G, and let G connecting lines GA and GD be

<sup>11</sup>Prop. 20 is often omitted in modern editions of *Elements*, as it is considered a later addition, and a direct consequence of proposition 19.

<sup>12</sup>“Outer” means neighbours in the sequence. Kochanski considers sequence of linear segments AB, BC, BD, DA, where both BC and BD are geometric means of their nearest neighbours in the sequence.

<sup>13</sup>A clear misprint: this should be DE.

*Demonstr.* Recta AD subtendit Angulum

210 rectum DEA. Igitur Circulus AGDE Diame-  
tro AD descripsit transit per verticem Anguli  
recti DEA. *juxta Schol. Clavii ad 31. 3 Elem.*  
Est autem recta EFG perpendicularis ad Dia-  
metrum AD. Ergo *per 3. 3. Elem.* tota EG  
215 bifariam secatur in F. Triangula igitur AFG,  
AFE orthogonia in F, sunt *per 4.1. Elem.* in-  
vicem aequalia : Eademque de causa aequan-  
tur Triangula DFG. DFE, ac proinde & totum  
DGA toti DEA aequale. Jam sic. In Orthogo-  
220 nio AED (par ratio de aequali AGD) ab angu-  
lo recto E cadit perpendicularis EF in basin  
AD: Ergo *per Coroll. 8.6. Elem.* Propor-  
tionales sunt tres AF.AE.AD. Sed eadem de  
causa in Orthogonio ADC duabus postremis  
225 e praecedenti serie, videlicet AE. AD. propor-  
tionalis est tertia AC. Igitur omnes quatuor  
AF.AE.AD.AC sunt in continua Proportione  
intra Circulum ADC, q.e.d.

### Corollarium.

230 Non difficulter hinc elicitur, easdem qua-  
tuor continuas in Spaerae quoque concipi pos-  
se, non modo praedicta, sed & alia ratione:  
Fingamus enim Sphaera ADC auferri Segmen-  
235 tum AHDA, cujus basis erit Circulus a Dia-  
metro AD, cujus meditas esto AGDA. Ductis  
autem Orthogonalibus DE. EF. in plano Cir-  
culi Sphaerae maximi ADC, nec non Orthogo-  
nali FG, in altero plano Semicirculi AGD, hoc  
240 est basi Segmati AHDA, jungatur AG: erunt  
enim ut antea, quatuor AF. AF, AD.AC. con-  
tinue proportionales, id patet e praecedenti  
discursu, qui non difficulter huc applicari po-  
terit, licet plana Circulorum ADG. AGD. sint  
245 diversa, & ad rectos invicem collocata.

Notandum vero est, Theorema praece-

made.

*Proof.* Line AD extends beneath the right  
angle DEA. Therefore the circle AGDE deter-  
215 mined by the diameter AD passes through the  
vertex of the right angle DEA, *according to com-  
mentary of Clavius to prop. 31 of book 3 of Ele-  
ments.* The straight line EFG is in fact perpen-  
dicular to the diameter AD. Therefore by *prop.*  
3, *book 3 of Elements*, the entire EG is divided  
220 into two equal parts at F. The triangles AFG,  
AFE having straight angle at F, are by *Prop.*  
4.1 *Elem.* equal to each other. For the same  
reason triangles DFG, DFE are equal, and hence  
the whole [triangle] DGA is equal to the whole  
225 [triangle] DEA. Now in the right-angled trian-  
gle AED (similarly reasoning applies to AGD)  
from the right angle E a perpendicular line EF  
falls onto the base AD: therefore, by *Coroll. 8.*  
6. *Elem.*, there are three proportionals AF, AE,  
230 AD. But for the same reason, in the right-angled  
triangle ADC, AC is proportional to the two last  
lines from the aforementioned sequence [of pro-  
portionals], namely AE and AD. Therefore all  
four AF, AE, AD, AC are proportional in suc-  
235 cession within the circle ADC, Q.E.D.

### Corollary.

Form there it is not difficult to elicit that  
the same four successive proportionals can  
240 be devised in a sphere, not by the preceding  
method, but by a different reasoning: let us  
namely imagine that a segment<sup>14</sup> AHDA is  
taken from a sphere ADC, whose basis is a circle  
with diameter AD, and whose one half is AGDA.  
245 Drawing perpendicular lines DE, EF in the  
plane of the great circle ADC, and perpendicular  
line GF, in another planar semicircle AGD, that  
is, in the base of the segment<sup>14</sup> AHDA, let they  
be joined by AG: there will be, as before, four  
250 successive proportionals AF, AG, AD, AC. It  
stands clear from the previous discourse, which  
could be applied here without difficulty, that  
it is permitted that planes of circles ADG and

<sup>14</sup>spherical cup

dens, loquendo pressius, non tam Semicirculo,  
quam Orthogonio cuilibet in similia subdiviso  
convenire: quia tamen ejus Demonstratio  
250 sequentibus inserviet Problematibus, visum  
est illud hoc loco tantisper indulgere Circulo,  
sine quo illa absolvi non possunt.

### PROPOSITIO III. Problema.

255 Inter duas datas, duas medias  
in continua ratione, duobus tan-  
tum digitis reperire.

Celeberrimum illud Problema Deliacum  
quot & quanta totius Orbis eruditi exercuerit  
260 ingenia, Geometris est notissimum, ut &  
variae illius absolvendi Praxes Organicae,  
a compluribus excogitatae, quarum aliae  
aliis sunt operosiores: Nostra haec videri  
poterit nonnihil Paradoxa, quod duobus  
265 tantum digitis unius manus, absolvatur, cum  
nonnullae requirant, & occupent utramque.

Datae sint, in *Figura VI.* duae AC. AB.  
quas inter duae mediae quaeruntur. In com-  
muni utriusque termino A figatur Regula AZ,  
270 instructa Cursore FY, qui semper insistat ad  
rectos ipsi regulae AZ, idque firmiter, ubicun-  
que collocetur. In illa fumatur AF, qualis  
datae AB, minori altera AC. Descripto au-  
tem super tota AC Semicirculo ADC in plano  
275 quopiam verticali, hoc est ad Horizontem rec-  
to, cui aequidistet Diameter AC: applicetur  
ad Peripheriam ADC Stylus quidam gracilis  
DS, e quo deorsum propendeat filum subtile  
cum appenso Pondere X, vel certe hujus loco  
280 regula quaedam sub gravis, accurate tamen  
aequilibrata: Nam si Stylus DS duobus digi-  
tis apprehensus pedetentim promoveatur per  
Circumferentiam AD, usque dum Perpendicu-  
lum DE cum Cursore FE sese mutuo interse-  
285 cent alicubi in recta AC, velut in Puncto E:  
istud probe notatum offeret quatuor Propor-  
tionales, quarum duae AE. AD. inter datas

AGD are different, and placed at right angles to 255R  
each other.

One must observe, however, that the previous  
theorem, speaking more precisely, is not as tied  
to a semicircle as to an arbitrary triangle sub-  
divided into similar ones: yet because its proof 260R  
lends itself to the following problems, certain le-  
niency toward the circle appeared in this place,  
without which they could to be brought out.

### Proposition III. Problem.

265R Between two given [quantities],  
find two means in successive pro-  
portions, with only two fingers.

It is very well known to Geometers how  
many and how great learned [men] exercised 270R  
[their] talents on this most famous Delian  
problem, and how various practical methods of  
its solution utilizing mechanical instruments,  
devised by many, have advantage one over  
another. With our method one could these as 275R  
paradoxes, because it utilizes only two fingers of  
one hand, while some other [methods] require  
and occupy both [hands].

Given are, as in *Figure VI*, two [quantities]  
AC and AB, between which two means are 280R  
sought. At their common end A a ruler AZ is  
fixed, equipped with cursor FY, which always  
firmly stands at the right angle to the ruler AZ,  
wherever placed. With the ruler the distance  
AF is taken, equal to the given AB, smaller 285R  
than AC. Somewhere in a vertical plane, that  
is, perpendicular to the horizontal line, let a  
semicircle ADC be drawn over the entire AC,  
whose diameter is equal to AC. Let at the  
circumference of ADC a thin stylus DS be 290R  
placed, from which a fine string is hanging down  
with a weight X attached, and at which finally  
the ruler is placed in equilibrium under gravity.  
For if the stylus DS is carefully held with two  
fingers and moved through the circumference 295R  
AD, all the way until the perpendicular DE  
intersects with the cursor FE somewhere on  
the straight line AC, for instance at point E,



extremas AF hoc est AB nec non AC interpo-  
nentur.

290 Demonstratio Problematis hujus, quoad  
rem, eadem est; cum adducto praecedenti  
Theoremate.

#### PROPOSITIO IV. Problema.

295

Id ipsum aliter, una Circini  
apertura.

Quoniam praecedens praxis ob situm pla-  
ni verticalem, & usum Perpendiculari, nonni-  
hil impedita cuiuspiam videri possit, quin & a  
300 Geometriae moribus aliena, dabimus alteram,  
praedictis incommodis haud obnoxiam.

Positis iisdem, in locum perpendiculari DEX  
Figurae praecedentis, subrogetur *in hac Figu-  
ra VII*. Parallelogrammum materiale KLMN,  
cujus unum Latus KL. fixum sit in plano, al-  
terum vero MN mobile, semper tamen ad rec-  
tos ipsi AC. Nam si tota AC divisa bifariam  
in O Circini pes unus figatur in O, alter au-  
tem intervallo OC diductus, tamdiu in Ar-  
cu CD provehatur, impellatque binas Regulas  
AZ. & MN in puncto intersectionis D, usque  
310 dum regula MN Cursorem FY secet in puncto  
E, posito in recta AC, obtinebuntur eaedem  
mediae AE. AD. longe commodiori ratione,  
315 quam fuerit praecedens; quae tamen ipsa, si  
Figura magnae molis fuerit, in vasto quopiam  
pariete usui esse poterit Architectis.

Non est cur hoc loco moneam de circi-  
320 no, ejusque Cruribus in regula quapiam mo-  
bilibus, nec de acie pedis alterius, quae re-  
gulas in puncto D subtiliter impellere debet;  
nec denique de nisu quodam regularum AZ,  
MN contra Circinum; hunc enim vel ipsa-  
rum regularum pondere, vel Elatare quopiam,  
325 aut unius digiti impulsu consequetur ingenio-  
sus quivis, ac istis etiam nostris longe meliora  
excogitabit.

this, as it rightly to be noted, produces four  
proportionals, of which two, AE and AD, are  
interposed between two given outward [lines]  
AF (that is AB) and AC.

300R

Demonstration of [correctness of solution of]  
this problem follows from the preceding proved  
theorem.

305R

#### PROPOSITION VI. Problem.

The same thing differently, with  
one aperture of the compass.

310R

Because the previous method seems to be  
hindered by the positioning of the vertical plane  
and by the use of the plumbline, which is alien  
to customs of geometers, we will give another  
[method], not burdened with the aforementioned  
315R inconveniences.

Setting up things as before, in place of the  
perpendicular DEX of the preceding figure, let  
in *Figure VII* a material parallelogram KLMN  
be substituted, whose one side KL is fixed in  
the plane, and another [side] MN is mobile, still  
always staying perpendicular to AC. For if the  
whole AC is divided into two [equal] parts at  
O, and one foot of the compass is placed at O,  
and one foot of the compass is placed at O,  
and another draws a line with [the aperture of]  
the interval AO, as long as it moves along the  
arc CD, let two rules AZ and MN, intersecting  
at the point D, be pushed until the ruler MN  
intersects the cursor FY at point E, positioned on  
the line AC. By this the same means AE and AD  
320R will be obtained, with a much more convenient  
method than the preceding one. This method, if  
the Figure was much larger, [placed] somewhere  
on a huge wall, could be of use to architects.

320R

325R

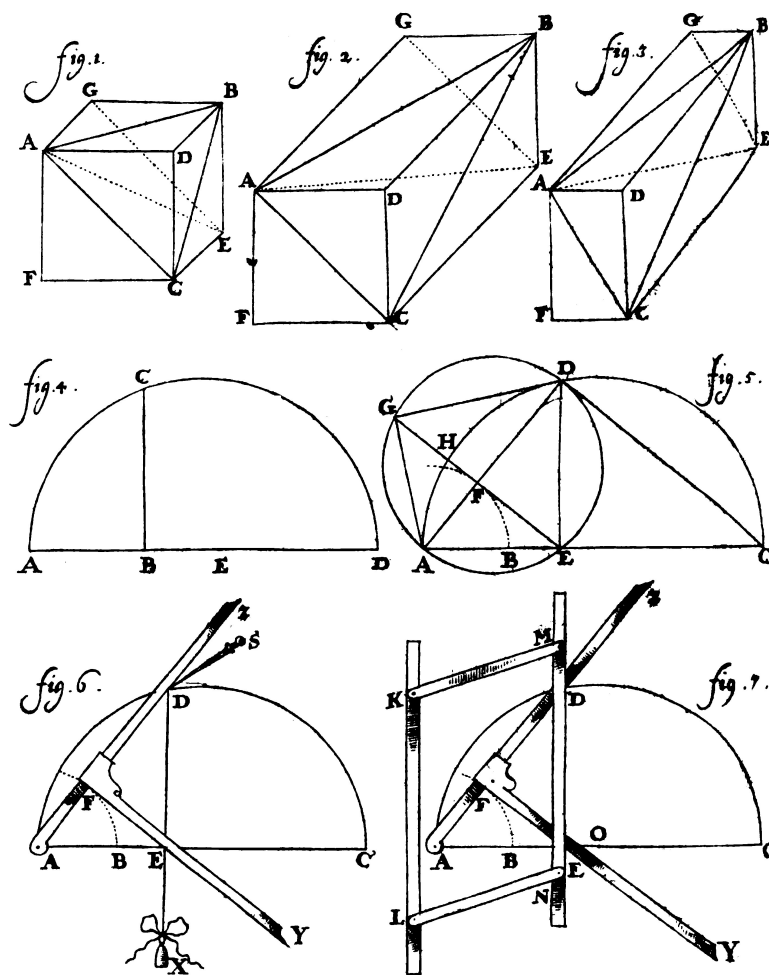
330R

It is not a place for me to give advise about  
the compass and its legs moveable is a certain  
ruler, or about the sharpness of the other leg,  
which should delicately push rules at point D, or  
finally about the pressure of rules AZ and MN  
against the compass. This indeed someone in-  
genious will attempt to achieve by the weight of  
rulers, or by some spring, or by the push of one

335R

340R

finger, or will devise something much better than that.



Figures 1 – 7.

# Appendix - list of propositions from *Elements* mentioned in the text of *Solutio theorematum*

*The first number indicates proposition number, the second one is the book number. Latin text from [3], English translation of propositions from [2]. Pappus generalization of 47.1 and Clavius' scholium for Prop. 31.3 translated by H.F.*

## Prop. 4.1

330 Si duo triangula duo latera duobus lateribus  
aequalia habeant, alterum alteri; habeant au-  
tem et angulum angulo aequalem, qui aequa-  
libus rectis lineis continentur: et basim basi  
aequalem habebunt; et triangulum triangulo  
335 aequale erit; et reliqui anguli reliquis angulis  
aequales, alter alteri, quibus aequalia latera  
subtenduntur.

## Prop. 47.1

340 In rectangulis triangulis, quod a latere rectum  
angulum subtendere describitur, quadratum  
aequale est quadratis quae a lateribus rectum  
angulum continentibus describuntur.

## Prop. 12.2

345 In obtusangulis triangulis quadratum ex la-  
tere obtusum angulum subtendere, majus est  
quam quadrata ex lateribus obtusum angu-  
lum continentibus, rectangulo contento bis ab  
uno laterum quae sunt circa obtusum angu-  
lum in quod productum perpendicularis cadit,  
350 et recta linea intercepta exterius a perpendi-  
culari ad angulum obtusum.

## Prop. 13.2

355 In omni triangulo, quadratum ex latere  
acutum angulum subtendere, minus est  
quam quadrata ex lateribus angulum illum  
continentibus, rectangulo contento bis ab uno  
laterum quae sunt circa acutum angulum,  
in quod productum perpendicularis cadit,  
et recta linea intercepta a perpendiculari ad  
360 angulum acutum.

## Prop. 3.3

365 Si in circulo recta linea per centrum ducta,  
rectam lineam quandam non ductam per cen-  
trum bifariam secet; et ad angulos rectos ip-  
sam secabit quod si ad angulos rectos ipsam  
secet, et bifariam secabit.

## Prop. 4.1

345R

If two triangles have the two sides equal to two  
sides respectively, and have the angles contained  
by the equal straight lines equal, they will also  
have the base equal to the base, the triangle will  
be equal to the triangle, and the remaining an-  
350R gles will be equal to the remaining angles respec-  
tively, namely those which the equal sides sub-  
tend.

## Prop. 47.1

In right-angled triangles the square on the side  
355R subtending the right angle is equal to the squares  
on the sides containing the right angle.

## Prop. 12.2

In obtuse-angled triangles the square on the side  
subtending the obtuse angle is greater than the  
360R squares on the sides containing the obtuse angle  
by twice the rectangle contained by one of the  
sides about the obtuse angle, namely that on  
which the perpendicular falls, and the straight  
line cut off outside by the perpendicular towards  
365R the obtuse angle.

## Prop. 13.2

In acute-angled triangles the square on the  
side subtending the acute angle is less than the  
squares on the sides containing the acute angle  
370R by twice the rectangle contained by one of the  
sides about the acute angle, namely that on  
which the perpendicular falls, and the straight  
line cut off within by the perpendicular towards  
the acute angle.

375R

## Prop. 3.3

If in a circle a straight line through the centre  
bisects a straight line not through the centre, it  
also cuts it at right angles; and if it cut at right  
angles, it also bisects it.

380R

**Prop. 8.6**

Si in triangulo rectangulo, ab angulo recto ad  
 370 basim perpendicularis ducatur; quae ad per-  
 pendicularem sunt triangula, et toti, et inter  
 se sunt similia.

**Prop. 31.6**

In triangulis rectangulis figura rectilinea quae  
 sit a latere rectum angulum subtendere, ae-  
 375 quale est eis quae a lateribus rectum angu-  
 lum continentibus sunt, similibus et similiter  
 descriptis.

**Prop. 19.7**

Si quatuor numeri proportionales fuerint, qui  
 380 ex primo, & quarto fit, numerus, aequalis erit  
 ei, qui ex secundo, & tertio fit, numero. Et si,  
 qui ex primo, & quarto fit, numerus, aequalis  
 fuerit ei, qui ex secundo, & tertio fit, numero;  
 ipsi quatuor numeri proportionales erunt.

**Prop. 20.7**

Si tres numeri proportionales fuerint; qui sub  
 extremis continetur, aequalis est ei, qui a me-  
 dio efficitur: Et si, qui sub extremis contine-  
 tur, aequalis fuerit ei, qui a medio describitur;  
 390 ipsi tres numeri proportionales erunt.

**Prop. 4.11**

Si recta linea duabus rectis lineis se invicem  
 secantibus in communi sectione ad rectos an-  
 gulos insistat, etiam ducto per ipsas plano ad  
 395 rectos angulos erit.

**Prop. 7.12**

Omne prisma triangulem habens basim, divi-  
 ditur in tres pyramides aequales inter se, quae  
 triangulares bases habent.

**Pappus generalization of 47.1, as given by Clavius**

In omni triangulo, parallelogramma quaecun-  
 que super duobus lateribus descripta, aequa-  
 lia sunt parallelogrammo super reliquo late-  
 405 re constituto, cuius alterum latus aequale sit,  
 & parallelum rectae ductae ab angulo, quae  
 duo illa latera comprehendunt, ad punctum,  
 in quo conveniunt latera parallelogrammorum  
 lateribus trianguli opposita, si ad partes an-  
 410 guli dicti producantur.

**Prop. 8.6**

If in a right-angled triangle a perpendicular be  
 drawn from the right angle to the base, the trian-  
 gles adjoining the perpendicular are similar both  
 to the whole and to one another.

385R

**Prop. 31.6**

In right-angled triangles the figure of the side  
 subtending the right angle is equal to the sim-  
 ilar and similarly described figures on the sides  
 containing the right angle.

390R

**Prop. 19.7**

If four numbers be proportional, the number pro-  
 duced from the first and fourth will be equal to  
 the number produced from the second and the  
 third; and, if the number produced from the first  
 and fourth be equal to that produced from the  
 second and third, the four numbers will be pro-  
 portional.

395R

**Prop. 20.7** (stated in the commentary to prop.  
 19.7 in [2])

400R

If three numbers be proportional, the product of  
 the extremes is equal to the square of the mean,  
 and conversely.

**Prop. 4.11**

If a straight line be set up at right angles to  
 two straight lines which cut one another, at their  
 common point of section, it will also be at right  
 angles to the plane through them.

405R

**Prop. 7.12**

Any prism which has a triangular base is divided  
 into three pyramids equal to one another which  
 have triangular bases.

410R

**Pappus generalization of 47.1, as given by Clavius**

In every triangle, any parallelograms built on two  
 sides are equal to the parallelogram built on the  
 remaining side, whose other side is equal and  
 parallel to the straight line drawn from the angle  
 made by the two sides of the triangle to the point  
 of intersection of extensions of the sides of par-  
 allelograms opposite to the sides of the triangle.

415R

420R

### Scholium Clavii ad 31.3

415 Manifestum quoque est conversum huius  
theorematis. Hoc est, segmentum circuli, in  
quo angulus constitutus est rectus, semicir-  
culus est. Nam si esset maius, angulus in eo  
foret acutus; si minus, obtusus.

### Corollarium ad Prop. 8.6

420 Ex hoc manifestum est, perpendicularem  
quae in rectangulo triangulo ab angulo  
recto in basin demittitur, esse mediam  
proportionalem inter duo basis segmenta.

### Clavius' scholium for Prop. 31.3

It is also clear that the converse of this theorem  
is true. That is, a segment of a circle, in which  
a right angle is constituted, is a semicircle. For 425R  
is it was greater, the angle in it would be acute,  
if lesser, it would be obtuse.

### Corollary to Prop. 8.6

From this is clear that, if a right angled triangle  
a perpendicular be drawn from the right angle 430R  
to the base, the straight line so drawn is a mean  
proportional between the segments of the base.

## References

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